Accepted Manuscript

Fractals

Article Title:	Decoding of upper limb movement by fractal analysis of Electroencephalogram (EEG) signal
Author(s):	Hamidreza Namazi, Tirdad Seifi Ala, Vladimir Kulish
DOI:	10.1142/S0218348X18500810
Received:	28 March 2018
Accepted:	11 June 2018
To be cited as:	Hamidreza Namazi, Tirdad Seifi Ala, Vladimir Kulish, Decoding of upper limb movement by fractal analysis of Electroencephalogram (EEG) signal, *Fractals*, doi: 10.1142/S0218348X18500810
Link to final version:	https://doi.org/10.1142/S0218348X18500810

Decoding of upper limb movement by fractal analysis of Electroencephalogram (EEG) signal

Hamidreza Namazi[a], Tirdad Seifi Ala[b], Vladimir Kulish[c]

[a] *School of Engineering, Monash University, Selangor, Malaysia*

[b] *Biomedical Engineering Department, Amirkabir University of Technology, Tehran, Iran*

[c] *School of Engineering, Avantiaka University, Ujjain, India*

Abstract

Analysis of human movements is an important category of research in biomedical engineering, especially for the rehabilitation purpose. The movement of limbs is investigated usually by analyzing the movement signals. Less efforts have been made to investigate how neural that correlate to the movements, are represented in the human brain. In this research, for the first time we decode the limb movements by fractal analysis of Electroencephalogram (EEG) signals. We investigated how the complexity of EEG signal changes in different limb movements in motor execution (ME), and motor imagination (MI) sessions. The result of our analysis showed that the EEG signal experiences greatest level of complexity in elbow flexion and hand-close movements in motor execution (ME), and motor imagination (MI) sessions respectively. On the other hand, the lowest level of complexity of EEG signal belongs to hand-open and rest condition in motor execution (ME), and motor imagination (MI) sessions respectively. Employing fractal theory in analysis of bio signals is not limited to EEG signal, and can be further investigated in other types of human's bio signals in different conditions. The result of these investigations can vastly been employed for the rehabilitation purpose.

Keyword: Limb movement, Fractal analysis, Electroencephalogram (EEG) signals, Complexity, Rehabilitation.

Introduction

Investigating about human limb movements is a major research topic in rehabilitation science. In this way, scientists have employed different techniques to analyze limb movements. The reported works on analysis of human movements using functional principal component analysis (fPCA) [1], correlation analysis and regression modeling [2], kinematic analysis and statistical regression [3-4], fractal analysis [5], multiscale entropy analysis [6] and wavelet analysis [7] are noteworthy to be mentioned. Since human movements are organized by the brain, an interesting category of works belongs to relating the human movements to the brain activity. Similarly, some scientists works on this area. The works on fMRI analysis of the brain in patients with phantom limb pain [8], EEG signal analysis in goal-directed movement intention [9], detection of the intention to move upper limbs by analysis of EEG signal [10], decoding of hand direction movements by EEG and MEG analysis [11] are noteworthy to be mentioned.

Fractal theory can be used to study the scaling properties of the EEG signal. A fractal object is a set that shows a self-similar pattern at every scale [12]. How the similarity is divided between different segments of object can be quantified using scaling exponent. Fractals can be simple or complex, which are presented with integer or non-integer values [13]. In general, the fractal dimension should satisfy the Szpilrajn inequality [14]:

$$\aleph \geq D_T \tag{1}$$

Where $\aleph$ and D_T are fractal dimension and topological dimension respectively. The amount of complexity of a pattern can be indexed by fractal dimension. During years, scientists have employed fractal theory to analyze different time series and patterns in engineering and science. There are plenty of works reported in literature about the application fractal theory in biomedical engineering. The works that employed fractal theory for analysis of DNA [15-18], eye movement [19-20], face [21], heart rate [22], respiration signal [23-24], spider brain signal [25-26] and animal movement behavior in foraging [27] are noteworthy to be mentioned. Similarly, scientists have employed fractal analysis to investigate about non-linear structure of EEG signal in different conditions. The works on prediction of seizure onset [28], distinguishing Alzheimer patients [29], classifying brain activity of epilepsy patients during ageing [30], analyzing the speech-evoked Auditory Brainstem Responses (s-ABR) between binaural and monaural listening conditions [31], analysis of the difference between normal subjects and subjects with stuttering in speech evoked auditory brainstem response [32], analysis of the influence of stress [33], meditation [34], visual [35-36], and auditory [37] stimuli on EEG signal are noteworthy to be mentioned.

Beside all efforts done on investigating about the non-linear structure of EEG signal in different conditions, no study has been reported yet that analyzed the influence of limb movements on brain signals using fractal theory. Therefore, in this research we employ fractal theory to investigate how the complexity of EEG signal changes in case of different limb movements. For this purpose, first, we bring our methodology and then we present the data collection and analysis, and accordingly the obtained results from analysis. In the last section, we draw discussion about obtained results and provide some future works.

Method

Here we want to investigate the variations of the nonlinear structure of EEG signal between rest condition and six limb movement conditions in two sessions, which are motor execution (ME), and motor imagination (MI). The limb movements that were same in both sessions include elbow flexion/extension, forearm supination/pronation and hand open/close. For this purpose, we employ fractal dimension as the measure of time series complexity, where its greater values stand for a more-complex process.

We can compute the fractal dimension using several methods [38] that approximate scaling and detail from limits estimated from regression lines over log-log plots of size versus scale. In fact, the general fractal dimension can be defined using entropy concept. Considering the EEG signal with the maximum and minimum values of voltage, V_{max} and V_{min} respectively, the total range of signal can be divided into M bins, where the size of each bin is $\delta \in$:

$$M = \frac{V_{max} - V_{min}}{\delta \in} \tag{2}$$

The probability of occurrence in the i'th bin is:

$$w_i = \lim_{N \to \infty} \frac{M_i}{M} \tag{3}$$

In equation (3), M_i is the total number of occurrence in the i-*th* bin. Equation (3) can be written for a time series as:

$$w_i = \lim_{T \to \infty} \frac{t_i}{T} \tag{4}$$

In equation (4), t_i is the total time of occurrence in the i-*th* bin. T represents the total time.

The general form of fractal dimensions can be written as:

$$D_q = \lim_{\epsilon \to 0} \frac{1}{q-1} \frac{\log \sum_{i=1}^{N} w_i^q}{\log \epsilon} \quad (5)$$

where ϵ is the scaling factor.

In this research, we employ box-counting method for calculation of fractal dimension. In fact, fractal dimension, D, is estimated as the exponent of a power law:

$$D = \lim_{\epsilon \to 0} \frac{\log N(\epsilon)}{\log \epsilon} \quad (6)$$

Where fractal dimension is estimated from regression line of the log-log plot of number boxes used to fill (N), versus scaling factor (ϵ) that is called box size here.

Therefore, in our approach, we analyze the variations of the complex structure of EEG signal in rest and different limb movements, using fractal dimension.

Database and analysis

In this research, we used the open-access data on EEG signal, which was collected by Ofner et. al. [39] and is available in [40]. The data include the EEG signals from 15 healthy subjects (9 F, 6 M). The EEG data includes seven conditions: one rest condition and six limb movements conditions. The EEG recording in limb movements was done in two sessions: motor execution (ME), and motor imagination (MI). The limb movements that were same in both sessions include elbow flexion/extension, forearm supination/pronation and hand open/close. In all movements, subjects started the task at a neutral position, the hand half open, the lower arm extended to 120 degree and in a neutral rotation. It is noteworthy that in the ME session, subjects were instructed to execute sustained movements. In the MI session, subjects performed kinesthetic MI. In case of each session, the EEG data was recorded from 61 channels (g.tec medical engineering GmbH, Austria) in 10 runs, with 42 trials per run. Ofner et al. [39] did band-pass filtering on the recorded EEG data from 0.01 Hz to 200 Hz and sampled with 512 Hz.

We proceeded with the data analysis by computing the fractal dimension. It is noteworthy that computation of fractal dimension was based on the box counting method [41].

In case of statistical analysis, first, the computed values of the fractal dimension were tested for normality using Anderson–Darling test. We run Post-hoc Tukey HSD test to compare the mean value of fractal dimension of EEG signal between different limb movements and also rest conditions within motor execution (ME), and motor imagination (MI). 2-tailed paired t-test was chosen to check the difference between motor execution (ME) and motor imagination (MI) in case of different limb movements and rest conditions. The significance level is case of all statistical tests was considered as 0.05

Results

Here we bring the result of fractal analysis. The result of Anderson–Darling test indicates that the computed fractal exponents of EEG signal in case of different limb movements are normal. Figure 1 shows the fractal dimension of EEG signal in case of rest and six limb movements' conditions in motor execution (ME) session.

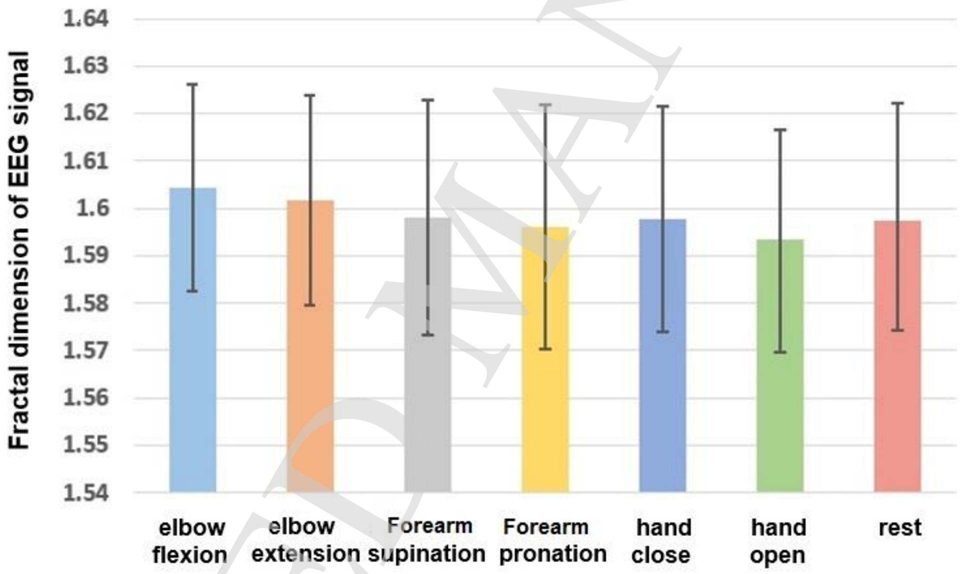

Figure 1. Fractal dimension of EEG signal in case of rest and six limb movement conditions in motor execution (ME) session.

As can be seen in Figure 1, the EEG signal in case of elbow flexion shows the greatest fractal dimension compared to other limb movements and rest condition. On the other hand, the fractal dimension of EEG signal for hand-open condition experiences the smallest value compared to other conditions. Since fractal dimension stands for the complexity of process, it can be said that the EEG signal in case of elbow flexion and hand-open movements has the greatest and lowest complexity. Another interesting result is the comparison between hand-open and hand-close conditions, where the hand-close movement causes greater fractal dimension of EEG signal than hand-open movement. In addition, as it is clear in this figure, the complexity of EEG signal in case of hand-open and forearm pronation movements is less than rest condition. The other comparisons between different limb movements show that the EEG signal in case of elbow flexion has greater fractal dimension than elbow extension. In addition, the EEG signal in case of forearm supination has greater fractal dimension than forearm pronation. As was mentioned before, we also run Post-hoc Tukey HSD test to compare the fractal dimension of EEG signal between each pair movements and also rest conditions. The result of this test indicates that there is no significant difference between fractal dimension of EEG signal between every pair of movements or rest.

In the similar analysis, Figure 2 shows the fractal dimension of EEG signal in case of rest and six limb movement conditions in motor imagination (MI) session.

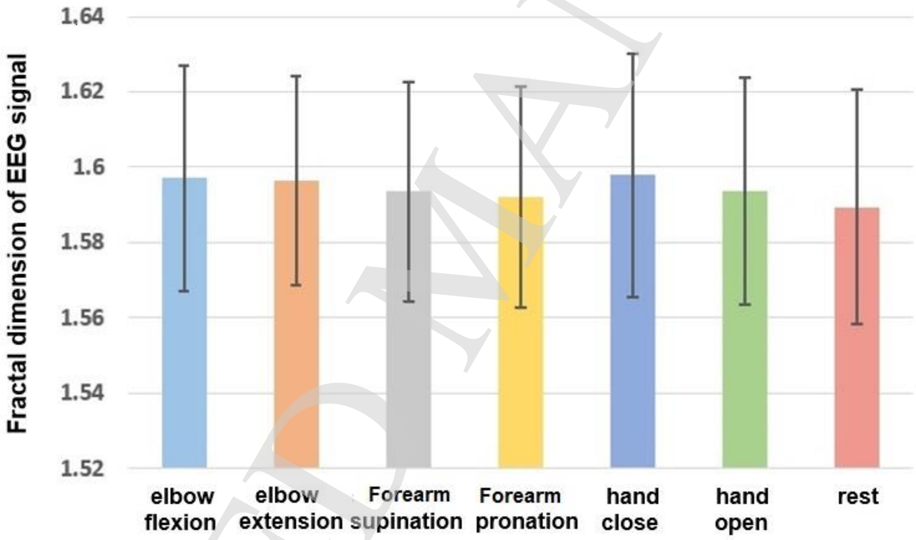

Figure 2. Fractal dimension of EEG signal in case of rest and six limb movement conditions in motor imagination (MI) session.

As can be seen in Figure 2, the EEG signal in case of hand-close movement shows the greatest fractal dimension compared to other limb movements and rest condition. On the other hand, the fractal dimension of EEG signal in rest condition experiences the smallest value compared to other conditions. Therefore, it can be said that the EEG signal in case of hand-open movement and rest has the greatest and lowest complexity. Similar to motor execution (ME) session, the EEG signal in case of elbow flexion has greater fractal dimension than elbow extension. Also, the EEG signal in case of forearm supination is greater than forearm pronation. In addition, the EEG signal in case of hand-close is greater than hand-open movement. Similar to the results in case of motor execution (ME), there is no significant difference between fractal dimension of EEG signal between every pair of movements or rest, based on Post-hoc Tukey HSD test.

As was mentioned before, we did 2-tailed paired t-test in order to compare the fractal dimension of EEG signal in case of different movements between motor execution (ME), and motor imagination (MI). The result of test is provided is Table 1.

Table 1. The result of 2-tailed paired t-test in comparison between motor execution (ME), and motor imagination (MI) in case of different limb movements.

Case of comparison	p-value
Elbow flexion	0.45
Elbow extension	0.57
Forearm supination	0.65
Forearm pronation	0.68
Hand close	0.97
Hand open	0.97

Based on Table 1, in case of fractal dimension of EEG signal for all limb movements, there is no significant difference between motor execution (ME), and motor imagination (MI) for different limb movements. Therefore, it can be said although the complexity of brain changes between motor execution (ME), and motor imagination (MI) sessions, however this difference is not significant. In addition, the smallest difference between motor execution (ME), and motor imagination (MI) was observed in case of hand close and also hand open conditions. On the other hand, the greatest difference between motor execution (ME), and motor imagination (MI) was observed in case of elbow flexion.

In summary, it can be said that fractal analysis could decode the difference between different limb movements and also rest condition, in both motor execution (ME) and motor imagination (MI) sessions.

Discussion

In this paper, we employed fractal theory in order to analyze human EEG signal in response to six limb movements and during rest. The experiments were done in two sessions that were motor execution (ME), and motor imagination (MI). In motor execution (ME), the greatest and smallest fractal dimension were obtained in case of elbow flexion and hand-open movements. Another interesting result in case of this session was that the fractal dimension of EEG signal in case of hand-open and forearm pronation movements was less than rest condition. The result of fractal analysis in case of limb movements in motor imagination (MI) session showed that the EEG signal in case of hand-close movement shows the greatest fractal dimension compared to other limb movements and rest condition. On the other hand, the fractal dimension of EEG signal in rest condition experiences the smallest value compared to other conditions. In addition, it is noteworthy to mention that in both sessions, the EEG signal in case of elbow flexion, forearm supination, and hand-open movement respectively has greater fractal dimension than elbow extension, forearm pronation, and hand-close movement.

We also did the statistical analysis using Post-hoc Tukey HSD test to compare the fractal dimension of EEG signal between each pair movements and also rest condition. In both motor execution (ME), and motor imagination (MI) sessions, the result of this test indicates that there is no significant difference in fractal dimension of EEG signal between every pair of movements or rest. In comparison between the obtained results for each limb movement between motor execution (ME), and motor imagination (MI), the statistical analysis (t-test) showed that there is no significant difference between motor execution (ME), and motor imagination (MI) in case of different limb movements. Therefore, it can be said although the complexity of brain changes between motor execution (ME), and motor imagination (MI) tasks, however this difference is not significant.

In general, it can be said that fractal analysis could decode the difference between different limb movements and also rest condition in both motor execution (ME) and motor imagination (MI) sessions. In fact, the analysis that was done in this research is one step forward to the studies [42-44] that only investigated the limb movements without linking them to the brain activity.

As it is known, the brain sends messages to the spinal cord and muscles through neural networks [45-46]. In addition, it is known that when the brain is engaged more with the processing, the EEG signal complexity (fractal dimension) changes more. Different limb movements were originated from different levels of activity of human brain. Therefore, it can be said that the EEG signals in case of different limb movements have different complexity levels (fractal dimension).

The application of fractal theory in analysis of EEG signal is not limited to analysis of the influence of limb movements. We can also extend our analysis in order to investigate the influence of different actions of human such as walking and respiration on his EEG signal. In further steps, we can also investigate how the complexity of different bio-signals are correlated with the complexity of EEG signal. For instance, we can study how the complexity of human gait time series is correlated with the complexity of EEG signal during walking. In addition, the obtained results can be mathematically linked to the brain activity using the current developed models in the literature [47-50], to generate the model of brain reaction to different limb movement. In overall, all these analysis can help scientists to link between human actions and brain activity that is the most important issue in rehabilitation science. On the hand, the developed model can be applied to other branch of engineering and science [51], where relating the memory and complexity of system is the important concept

References

[1] Averta, G., Santina, C. D., Battaglia, E., Felici, F., Bianchi, M. & Bicchi, A. Unvealing the Principal Modes of Human Upper Limb Movements through Functional Analysis. Front. Robot. AI; DOI: 10.3389/frobt.2017.00037 (2017).

[2] Moro, M., Goparaju, B., Castillo, J., Alameddine, Y. & Bianchi, M. Periodic limb movements of sleep: empirical and theoretical evidence supporting objective at-home monitoring. Nature and Science of Sleep. 8, 277-289; DOI: 10.2147/NSS.S101753 (2016).

[3] Christovão, F. G. et al. Three-dimensional kinematic analysis of head and limb movements of lame and non-lame colts. Equine and comparative exercise physiology. 4 (1), 31-36 (2007).

[4] Tascau, M. T., Balanean, F. & Krepelka, M. Assessment and Prediction of Inter-joint Upper Limb Movement Correlations based on Kinematic Analysis and Statistical Regression. AIP Conf. Proc. 1558, 1648-1651; DOI: 10.1063/1.4825844 (2013).

[5] Hunt, N., McGrath, D. & Stergiou, N. The influence of auditory-motor coupling on fractal dynamics in human gait. Sci. Rep. 4, 5879; DOI:10.1038/ srep05879 (2014).

[6] Costa, M., Peng, C. K., Goldberger, A. L. & Hausdroff, J. M. Multiscale entropy analysis of human gait dynamics. Physica A: Statistical Mechanics and its Applications. 330 (1-2), 53-60 (2003).

[7] Singh, A. & Thakur, A. Human gait analysis using wavelet de-noising and total variation filtering. 2015 International Conference on Green Computing and Internet of Things (ICGCIoT), Noida, 2015, 704-708; DOI: 10.1109/ICGCIoT.2015.7380554.

[8] Lotze, M., Flor, H., Grodd, W., Larbig, W. & Birbaumer, N. Phantom movements and pain. An fMRI study in upper limb amputees. Brain. 124 (Pt 11), 2268-77 (2001).

[9] Pereira, J., Ofner, P., Schwarz, A., Sburlea, A. I. & Müller-Putz, G. R. EEG neural correlates of goal-directed movement intention. Neuroimage. 1 (149), 129-140; DOI: 10.1016/j.neuroimage.2017.01.030 (2017).

[10] Gudiño-Mendoza, B., Sanchez-Ante, G. & Antelis, J. M. Detecting the Intention to Move Upper Limbs from Electroencephalographic Brain Signals. Comput. Math. Methods Med. 2016, 3195373; DOI: 10.1155/2016/3195373 (2016).

[11] Waldert, S. et al. Hand Movement Direction Decoded from MEG and EEG. Journal of Neuroscience. 28 (4), 1000-1008; DOI: 10.1523/JNEUROSCI.5171-07.2008 (2008).

[12] Namazi, H. Can we Mathematically Correlate Brain Memory and Complexity? ARC Journal of Neuroscience. 3 (2), 1-3; DOI: 10.20431/2456-057X.0302003 (2018).

[13] Namazi, H., Akhavan Farid, A., & Teck Seng, C. Fractal based analysis of the influence of cutting depth on complex structure of cutting forces in rough end milling. DOI: 10.1142/S0218348X18500688 (2018).

[14] Kulish, V.V. Partial Differential Equations (Pearson, 2010).

[15] Namazi, H. & Kiminezhadmalaie, M. Diagnosis of Lung Cancer by Fractal Analysis of Damaged DNA. Comput. Math. Methods Med. 2015, DOI:10.1155/2015/242695 (2015).

[16] Namazi, H., Kulish, V. V., Delaviz, F. & Delaviz, A. Diagnosis of skin cancer by correlation and complexity analyses of damaged DNA. Oncotarget, 6 (40), 42623-31. DOI: 10.18632/oncotarget.6003 (2015).

[17] Namazi, H. Can We Explain the Memory Transfer between Generations by Mathematical Analysis of DNA Walk?. ARC Journal of Neuroscience. 2(2), 1-3; DOI: 10.20431/2456-057X.0202001 (2017).

[18] Namazi, H. et al. Mathematical Modelling and Prediction of the Effect of Chemotherapy on Cancer Cells. Sci. Rep. 5, 13583; DOI: 10.1038/srep13583 (2015).

[19] Namazi, H., Daneshi, A., Azarnoush, H., Jafari, S. & Towhidkhah, F. Fractal based analysis of the influence of auditory stimuli on eye movements. Fractals. DOI: 10.1142/S0218348X18500408 (2018).

[20] Namazi, H. et al. The analysis of the influence of fractal structure of stimuli on fractal dynamics in fixational eye movements and EEG signal. Sci. Rep. 6, 26639; DOI: 10.1038/srep26639 (2016).

[21] Namazi, H. et al. The fractal based analysis of human face and DNA variations during aging. Biosci. Trends. 10; DOI: 10.5582/bst.2016.01182 (2016).

[22] Namazi, H. & Kulish, V.V. Fractal Based Analysis of the Influence of Odorants on Heart Activity. Sci. Rep. 6, 38555; DOI: 10.1038/srep38555 (2016).

[23] Namazi, H., Akrami, A. & Kulish, V. V. The Analysis of the Influence of Odorant's Complexity on Fractal Dynamics of Human Respiration. Sci. Rep. 6: 26948, 1-8 (2016).

[24] Namazi, R. H. Fractal-based analysis of the influence of music on human respiration. Fractals, 25. DOI: 10.1142/S0218348X17500591 (2017).

[25] Namazi, H. Can We Correlate the Spider's Brain Activity to it Spinning Web Activity?. ARC Journal of Neuroscience. 2(1), 17-18; DOI: 10.20431/2456-057X.0201005 (2017).

[26] Namazi, R. H. The Complexity Based Analysis of the Correlation between Spider's Brain Signal and Web. ARC Journal of Neuroscience. 2 (4), 38-44; DOI: 10.20431/2456-057X.0203006 (2017).

[27] Namazi, H. R. Fractal Based Analysis of Movement Behavior in Animal Foraging. ARC Journal of Neuroscience. 2 (3), 1-3; DOI: 10.20431/2456-057X.0203001 (2017).

[28] Namazi, et al. A signal processing based analysis and prediction of seizure onset in patients with epilepsy. Oncotarget 7, 342-350; DOI: 10.18632/oncotarget.6341 (2016).

[29] Staudinger, T. & Polikar, R. Analysis of complexity based EEG features for the diagnosis of Alzheimer's disease. Conf. Proc. IEEE Eng. Med. Biol Soc. 2011, 2033-6; DOI: 10.1109/IEMBS.2011.6090374 (2011).

[30] Namazi, H. & Jafari, S. Age-based variations of fractal structure of EEG signal in patients with epilepsy. Fractals; DOI: 10.1142/S0218348X18500512 (2018).

[31] Mozaffarilegha, M., Namazi, H. R., Ahadi, M. & Jafari, S. Complexity based analysis of the difference in speech-evoked Auditory Brainstem Responses (s-ABR) between binaural and monaural listening conditions. Fractals; DOI: 10.1142/S0218348X18500524 (2018).

[32] Mozaffarilegha, M., Namazi, H. Tahaei, A. A. & Jafari, S. Complexity based analysis of the difference between normal subjects and subjects with stuttering in speech evoked auditory brainstem response. Journal of Medical and Biological Engineering. 2018. In press.

[33] Sourina, O., Wang, Q., Liu, Y. & Nguyen, M. K. Fractal-Based Brain State Recognition from EEG in Human Computer Interaction. In: Fred A., Filipe J., Gamboa H. (eds) Biomedical Engineering Systems and Technologies. BIOSTEC 2011. Communications in Computer and Information Science, 273. Springer, Berlin, Heidelberg, 2013.

[34] Kaur, C. & Singh, P. EEG Derived Neuronal Dynamics during Meditation: Progress and Challenges. Adv. Prev. Med. 2015, 614723; DOI: 10.1155/2015/614723 (2015).

[35] Namazi, H. & Kulish, V. V. Fractional Diffusion Based Modelling and Prediction of Human Brain Response to External Stimuli. Comput. Math. Methods Med. 2015, 148534; DOI:10.1155/2015/148534 (2015).

[36] Ahmadi-Pajouh, M. A., Seifi Alaa,T., Zamanian, F., Namazi, H. & Jafari, S. Fractal-based classification of human brain response to living and non-living visual stimuli. DOI: 10.1142/S0218348X1850069X (2018).

[37] Namazi, H. et al. Analysis of the influence of memory content of auditory stimuli on the memory content of EEG signal. Oncotarget, 7, 56120-56128; DOI: 10.18632/oncotarget.11234 (2016).

[38] Namazi, H. Can We Study the Correlation between Human Brain Signal and Other biological Signals?. ARC Journal of Neuroscience. 2 (2), 7-9; DOI: 10.20431/2456-057X.0202003 (2017).

[39] Ofner, P., Schwarz, A., Pereira, J. & Müller-Putz, G. R. Upper limb movements can be decoded from the time-domain of low-frequency EEG. PLOS ONE 12(8): e0182578 (2017).

[40] http://bnci-horizon-2020.eu/database/data-sets

[41] Namazi, H., Akrami, A., Haghighi, R., Delaviz, A. & Kulish, V.V. Analysis of the Influence of Element's Entropy on the Bulk Metallic Glass (BMG) Entropy, Complexity and Strength. Metallurgical and Materials Transactions A. DOI: 10.1007/s11661-016-3870-3 (2016).

[42] Elsworth-Edelsten, C. et al. Upper limb movement analysis during gait in multiple sclerosis patients. Hum. Mov. Sci. 54 (248-252); DOI: 10.1016/j.humov.2017.05.014 (2017).

[43] Turner-Stokes, L. & Reid, K. Three-dimensional motion analysis of upper limb movement in the bowing arm of string-playing musicians. Clin. Biomech (Bristol, Avon). 14 (6), 426-33 (1999).

[44] M. Zeng et al. Biomechanical Analysis of Typical Upper Limb Movements Based on Kinect-LifeMOD. Applied Mechanics and Materials. 599-601, 534-538 (2014).

[45] Namazi, H. & Kulish, V. V. A mathematical based calculation of a myelinated segment in axons. Comput. Biol. Med. 2013 Jul; 43(6):693-8. DOI: 10.1016/j.compbiomed.2013.03.005

[46] Seetharaman, K., Namazi, H. & Kulsih, V. V. Phase lagging model of brain response to external stimuli—modeling of single action potential. Comput. Biol. Med. 42, 857-862 (2012).

[47] Namazi, H. & Kulish, V. V. Mathematical Modeling of Human Brain Neuronal Activity in the Absence of External Stimuli. Journal of Medical Imaging and Health Informatics. 2 (4), 400-407 (2012).

[48] Namazi, H., Kulish, V. V., Wong, A. & Nazeri, S. Mathematical Based Calculation of Drug Penetration Depth in Solid Tumors. Biomed Res Int. 2016: 8437247; DOI: 10.1155/2016/8437247 (2016).

[49] Vevek, U. S., Namazi, H., Haghighi, R. & Kulish V. V. Analysis & validation of exact solutions to Navier-Stokes equation in connection with quantum fluid dynamics. Mathematics in Engineering, Science & Aerospace (MESA), 7 (20), 389-417 (2016).

[50] Namazi, H. & Kulish, V. V. A mathematical based definition of human consciousness. Mathematics in Engineering, Science & Aerospace (MESA). 3 (2), 189-198 (2012).

[51] Akhavan Farid, A., Sharif, S. & Namazi, H. Effect of Machining Parameters and Cutting Edge Geometry on Surface Integrity when Drilling and Hole Making in Inconel 718. SAE International Journal of Materials and Manufacturing; DOI10.4271/2009-01-1412 (2009).